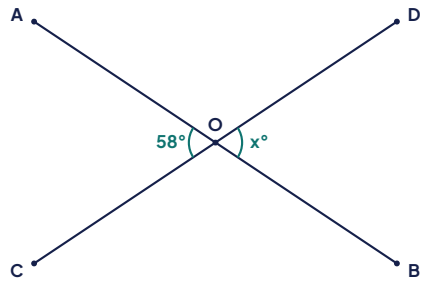
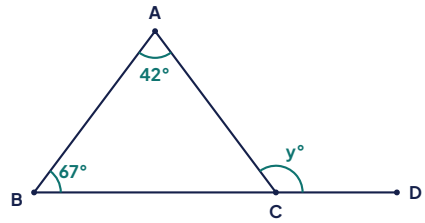
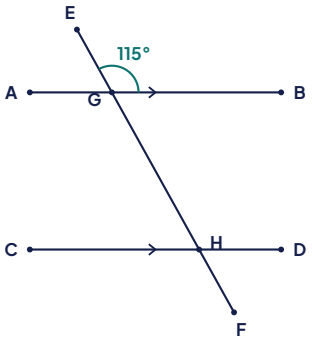
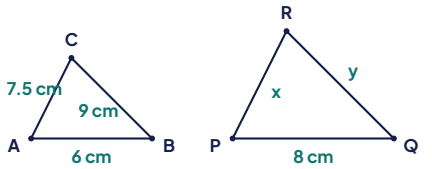
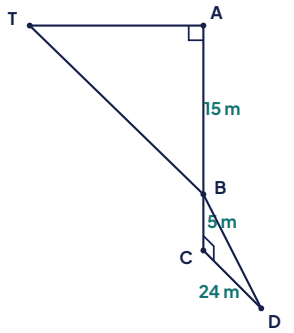
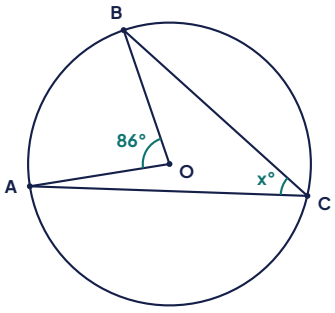
Geometry: IGCSE Maths worksheet

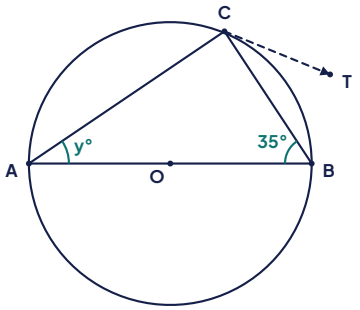
8 questions · 32 marks · Angle properties, Similarity and congruence, Circle theorems

Q	Answer and marking steps	Marks
1	In the diagram, lines AB and CD are straight lines crossing at point O. Angle AOC = 58°. Find angle BOD, giving a reason for your answer.  **Answer:** 58° (vertically opposite angles) • States angle BOD = 58° — 1 • Gives correct reason: vertically opposite angles — 1 Angle properties	**2**
2	The diagram shows a triangle ABC with angle A = 42° and angle B = 67°. The side BC is extended to point D, forming an exterior angle at C. (a) Find angle ACB. (b) Find the exterior angle ACD, giving a reason for your answer.  **Answer:** (a) 71° (b) 109° • (a) Uses angle sum of triangle = 180° — 1 • (a) Angle ACB = 180 – 42 – 67 = 71° — 1 • (b) Exterior angle = 180 – 71 = 109° (angles on a straight line sum to 180°) OR exterior angle = sum of two opposite interior angles — 2 Angle properties	**4**

Q	Answer and marking steps	Marks
3	Lines AB and CD are parallel. A transversal line EF crosses AB at point G and crosses CD at point H. Angle EGB = 115°. (a) Find angle GHD, giving a reason. (b) Find angle GHC, giving a reason.  **Answer:** (a) 115° (corresponding angles) (b) 65° (co-interior/allied angles, or angles on a straight line) • (a) Angle GHD = 115° 1 • (a) Correct reason: corresponding angles (parallel lines) 1 • (b) Angle GHC = 180 – 115 = 65° 1 • (b) Correct reason: angles on a straight line / co-interior angles 1 Angle properties	4
4	Triangles ABC and PQR are similar, with angle A = angle P, angle B = angle Q, and angle C = angle R. AB = 6 cm, BC = 9 cm, AC = 7.5 cm, and PQ = 8 cm. (a) Find the scale factor from triangle ABC to triangle PQR. (b) Find the length QR. (c) Find the length PR.  Not drawn to scale **Answer:** (a) 4/3 (b) QR = 12 cm (c) PR = 10 cm • (a) Scale factor = PQ ÷ AB = 8 ÷ 6 = 4/3 1 • (b) QR = BC × scale factor = 9 × 4/3 = 12 cm 1 • (c) PR = AC × scale factor = 7.5 × 4/3 = 10 cm 2 Similarity and congruence	4

Q	Answer and marking steps	Marks
5	Nadia wants to find the width of a river. She stands at point A on one bank and sights a tree T directly across the river. She then walks along the bank to point B, 15 m from A, and then to point C, 5 m further along the same line from B. From C, she walks away from the river until point D is in line with B and T. CD is measured as 24 m. Triangle ABT and triangle CBD are similar (angle A = angle C = 90°, and angle ABT = angle CBD as vertically opposite angles). AB = 15 m, BC = 5 m, CD = 24 m. (a) Explain why triangle ABT is similar to triangle CBD. (b) Find the width of the river, AT.  Not drawn to scale **Answer:** (a) Both have a right angle (at A and C), and the angles at B are vertically opposite and so equal; therefore the triangles are similar (AA). (b) AT = 72 m • (a) States both triangles have a right angle 1 • (a) States vertically opposite angles at B are equal, so AA similarity 1 • (b) Sets up ratio AT/CD = AB/BC 1 • (b) AT = 24 × (15/5) = 72 m 1 Similarity and congruence	4
6	In the diagram, O is the centre of the circle. Points A and B lie on the circle, and angle AOB = 86°. Point C also lies on the circle, on the major arc AB. Find angle ACB, the angle at the circumference subtended by the same arc AB.  **Answer:** 43° • Uses the theorem: angle at centre is twice the angle at circumference 1 • Angle ACB = 86 ÷ 2 = 43° 1 Circle theorems	2

Q	Answer and marking steps	Marks
7	In the diagram, O is the centre of the circle. AB is a diameter. C is a point on the circle, and angle ABC = 35°. TC is a tangent to the circle at point C. (a) Find angle ACB, giving a reason. (b) Find angle BAC. (c) Find angle BCT, the angle between the chord BC and the tangent at C, giving a reason.	6

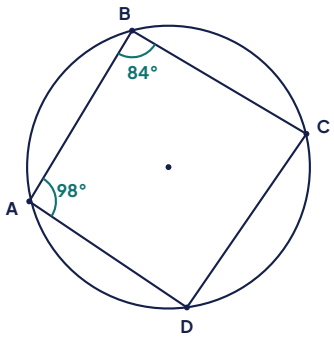


Answer: (a) 90° (angle in a semicircle is a right angle)
(b) 55°
(c) 55° (alternate segment theorem: angle BCT = angle BAC)

- (a) Angle ACB = 90° 1
- (a) Reason: angle in a semicircle is 90° 1
- (b) Angle sum of triangle: 180 – 90 – 35 = 55° 1
- (c) States alternate segment theorem 1
- (c) Identifies BCT equals angle in alternate segment (angle BAC) 1
- (c) Angle BCT = 55° 1

Circle theorems

Q	Answer and marking steps	Marks
8	In the diagram, ABCD is a cyclic quadrilateral. O is the centre of the circle. Angle DAB = 98° and angle ABC = 84°. (a) Find angle BCD, giving a reason. (b) Find angle ADC, giving a reason. (c) Explain, using circle theorems, why opposite angles of a cyclic quadrilateral must sum to 180°. (A brief explanation using arcs or angle at centre/circumference is sufficient.)	6



Answer: (a) Angle BCD = 82°
(b) Angle ADC = 96°
(c) Opposite angles of a cyclic quadrilateral stand on arcs that together make up the whole circle (360°). The angle at the centre for each arc is twice the angle at the circumference, so the two angles at the circumference add to half of 360°, which is 180°.

- (a) Angle BCD = 180 – 98 = 82° 1
- (a) Reason: opposite angles of a cyclic quadrilateral sum to 180° 1
- (b) Angle ADC = 180 – 84 = 96° 1
- (b) Reason: opposite angles of a cyclic quadrilateral sum to 180° 1
- (c) Mentions angle at centre is twice angle at circumference for each pair of opposite angles 1
- (c) Concludes the two arcs/centre angles total 360°, so circumference angles total 180° 1

Circle theorems